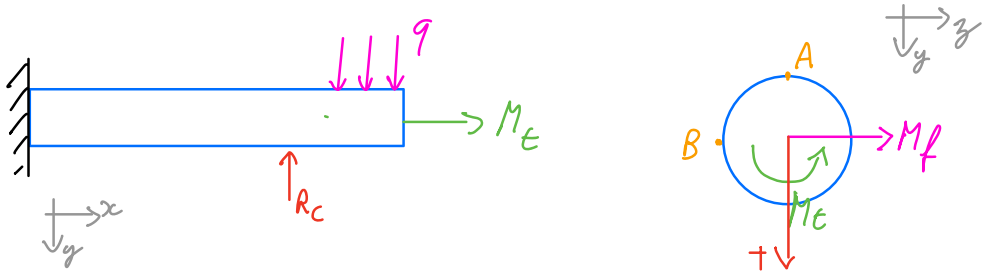
15.1 et 15.2



TRESCA : faut σ_1 et σ_3 en A et B

von MISES : σ_x σ_y σ_z τ_{xy} τ_{xz} τ_{yz} ou σ_1 σ_2 σ_3 en A et en B

TORSION $\quad \tau_{xz}(A) = -\tau_{xy}(B) = -\dfrac{M_t}{W_p}$ $\qquad W_p = \dfrac{I_p}{D/2} = \dfrac{\pi D^4/32}{D/2} = \dfrac{\pi D^3}{16}$

$$= -\frac{16\, M_t}{\pi D^3} = -57\ \text{MPa}$$

FLEXION

$$M_f = \int_{b-a}^{a} q \cdot x \cdot dx = \frac{1}{2} q (2ab - a^2)$$

$\sigma_x(y) = y\, M_f / I_z$ $\qquad$ B : $y = 0$ $\quad \sigma_x(B) = 0$

A : $y = D/2$ $\quad \sigma_x(A) = \dfrac{D}{2} \dfrac{M_f}{I} = \dfrac{32 M_f}{\pi D^3} = 204\ \text{MPa}$ $\quad I = \dfrac{\pi D^4}{64}$

$\sigma_y = 0$

EFFORT TRANCHANT

$T = qa$ $\qquad \tau_{xy}(y) = \dfrac{T\, S'(y)}{I\, b}$ $\qquad$ A, surface $\perp$ à T $\quad \tau_{xy}(A) = 0$

en B, $\quad \tau_B = \dfrac{T}{ID} \displaystyle\int_0^{D/2} \int_0^{\pi} r\,(r \sin\phi)\, d\phi\, dr$

$$\tau_B = \frac{16T}{3\pi D^2} = \frac{4T}{3F}$$

CONTRAINTES PRINCIPALES en A. → CHAP 3. CONTRAINTE BI-DIMENSIONELLE

$\sigma_3 = 0$

$$\sigma_{1\,3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} \quad \text{et } \sigma_2 = 0$$

$$= \frac{\sigma_x(A)}{2} + \sqrt{\frac{\sigma_x^2(A)}{4} + \tau_{xz}^2(A)} = 216 \text{ MPa}$$

$\sigma_2 = 0$

$$\sigma_3 = \frac{\sigma_x(A)}{2} - \sqrt{\frac{\sigma_x^2(A)}{4} + \tau_A^2} = -12 \text{ MPa}$$

TRESCA. en A

CONTRAINTE EQUIVALENTE en A, TRESCA

$$\sigma_G(A) = \sigma_1 - \sigma_3 = 228 \text{ MPa}$$

$$\eta(A) = \frac{\sigma_e}{\sigma_G} = \frac{295}{228} = 1.29$$

CONTRAINTES PRINCIPALES en B

$$\sigma_1 = +\tau = \frac{16 M_f}{\pi D^3} + \frac{4T}{3F} = 85 \text{ MPa}$$

$$\sigma_3 = -\tau = -85 \text{ MPa}$$

TRESCA. en B

CONTRAINTE EQUIVALENTE en B, TRESCA

$$\sigma_G(B) = \tau - (-\tau) = 2\tau = 170 \text{ MPa}$$

$$n(B) = \frac{\sigma_E}{\sigma_G} = 1.74$$

VON MISES

en A $\quad \sigma_G(A) = \frac{1}{\sqrt{2}} \sqrt{\sigma_x^2(A) + \sigma_x^2(A) + 6\tau_{xz}^2}$

$$\sigma_G(A) = 222 \text{ MPa}$$

$$n(A) = \frac{\sigma_e}{\sigma_G} = 1.33$$

en B $\quad \sigma_G(B) = \frac{1}{\sqrt{2}} \sqrt{6\tau_{xy}^2} = 147 \text{ MPa}$

$$n(B) = \frac{\sigma_e}{\sigma_G} = 2.01$$

$n_{\text{VON MISE}} > n_{\text{TRESCA}}$

↳ plus conservateur